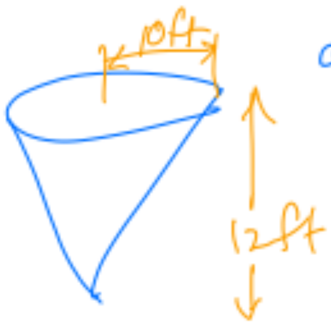
Application of Derivatives:

• Related rates problems

Example: A tank has the shape of an inverted cone with radius 10 ft and height 12 ft.

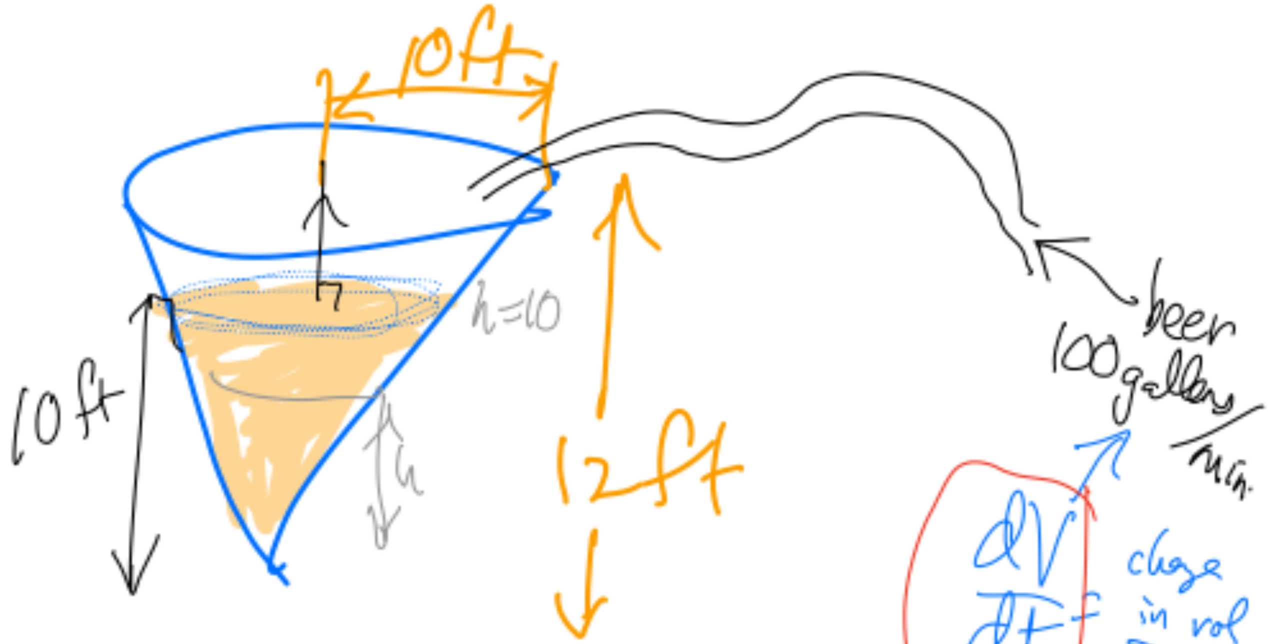


Billy Bob fills this tank with Bud Light with a hose. The hose pumps at a rate of 100 gallons per minute. When the height of beer is 10 ft from the bottom, how fast is the height rising?

Related Rates Problem:

- Draw picture ← put in variables
- What rate is given and what rate are you supposed to find?
- Make an equation involving the variables
- Take the derivative (often implicit diff.)

• Solve for the rate.



We want $\frac{dh}{dt} = ?$

$\frac{dV}{dt}$ = change in vol / change in t

side view

$V = \frac{1}{3}\pi r^2 h$

Volume of cone

$V = \frac{1}{3}\pi r^2 h$

We can make V only depend on h if we use the ratio

$$h\,\frac{10}{12} = \frac{r}{h}\,h$$

(similar triangles)

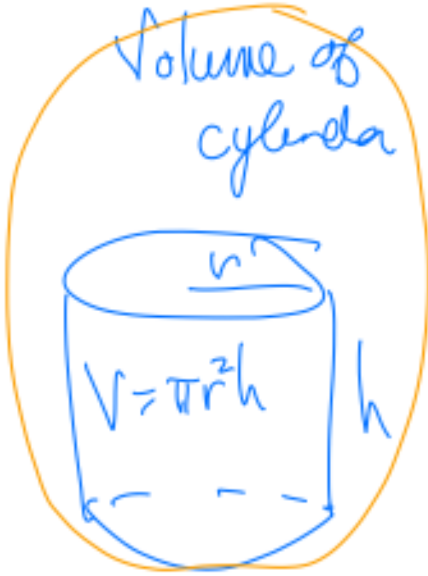
$$\Rightarrow r = \frac{10h}{12} = \frac{5h}{6}$$

$$\Rightarrow V = \frac{1}{3}\pi\left(\frac{5h}{6}\right)^2 \cdot h$$

$$V = \frac{1}{3}\pi\,\frac{25h^2 \cdot h}{36}$$

$$\Rightarrow \boxed{V = \frac{25\pi}{108}h^3}$$

relationship equation



Take derivative of both sides with respect to time:

$$\frac{dV}{dt} = \frac{25\pi}{108} \cdot 3h^2 \frac{dh}{dt}$$

$$\Rightarrow \boxed{\frac{dV}{dt} = \frac{25\pi}{36} \cdot h^2 \frac{dh}{dt}}$$

In our situation

$$\frac{dV}{dt} = \frac{100 \text{ galons}}{\text{min}} = \frac{100 \text{ gallons}}{\text{min}} \cdot \frac{1\ ft^3}{7.48 \text{ gallon}} = 13.3689\ ft^3/\text{min}$$

converts to ft^3/min.

$h = 10\,ft$

$\frac{dh}{dt} = ?$

$$\Rightarrow \boxed{\frac{dV}{dt} = \frac{25\pi}{36} \cdot h^2 \frac{dh}{dt}}$$

$$13.3689 \frac{ft^3}{min} = \frac{25\pi}{36}(10ft)^2 \frac{dh}{dt}$$

$$\Rightarrow \frac{36(13.3689)ft^{\cancel{3}\,1}}{25\pi\, 100\cancel{ft^2}\, min} = \frac{dh}{dt}$$

$$\Rightarrow \frac{.06127\,ft}{min} = \frac{dh}{dt}$$

$$\Rightarrow \frac{(.06127)\cancel{ft}}{min} \frac{12in}{\cancel{ft}} = \frac{dh}{dt}$$

$$\approx .7352 \frac{in}{min} \approx \boxed{.75\, in/min}$$

Question:

A puddle under Ken's truck starts growing at a rate of 3 inches per second. The puddle is in the shape of a circle (disk), currently 12 inches in diameter. How fast is the area of the puddle growing?

$$\frac{dD}{dt} = \frac{3\text{ in}}{\text{sec}}$$

$$A = \text{Area} = \pi r^2 = \pi\left(\tfrac{1}{2}D\right)^2$$

$$\boxed{A = \frac{\pi}{4}D^2.}$$

$$\frac{dA}{dt} = ?$$

$$\frac{d}{dt}:$$

$$\frac{dA}{dt} = \frac{\pi}{4}\cdot 2D\cdot\frac{dD}{dt}$$

$$\frac{dA}{dt} = \frac{\pi}{\cancel{4}}\cdot 2\left(\overset{3}{\cancel{12}}\right)(3)\ \frac{\text{in}^2}{\text{sec}}$$

$$= 18\pi\ \text{in}^2/\text{sec}$$

$= \boxed{56.5 \ in^2/sec}$.